The arches of the facade of the Palau Güell. Hypothesis about its conformation

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SUMMARY

The twin arches of the façade of the Palau Güell in Barcelona have been classified in the bibliography on Gaudí as having a catenary or parabolic profile. This assignment is based on a certain similarity that these curves have at first glance. Recent studies have shown that they are Rankine curves actually. This paper investigates the functional, symbolic and mechanical reasons on which the choice of this form of complicated layout would have been based and analyzes the graphic procedures and other methods to which Gaudí may have resorted for the design of these arches, with a foundation in those used in other works.

The profile of the arches on the façade of the Palau Güell in Barcelona was a subject generally contemplated in a superficial way in the profuse Gaudinian bibliography where its unusual form was generally classified without scientific rigor. The same treatment received many other forms used by the architect in his works.

Referred to the arches used by Gaudí and seeking clarity in the matter, we carried out an investigation some years ago. On the basis of frontally taken photographs and by means of the application of a graphic procedure we investigated in what occasions the arches are parabolas or catenaries, and when they do not respond to any of the two mentioned profiles.[1]

In the case that concerns us in this article, the Palau Güell portals, the arches fit into this last category: they are not parabolic nor catenaries, which is not a surprise since the simple visual inspection reveals that their profile moves away from those curves.

The publication im 2016 and 2017 of two studies by researchers at the Universitat Rovira i Virgili of Tarragona brought this issue into consideration.[2] They consisted in the development of a method to determine with precision the shape of an arc, its application to the arches of the access to the Palau Güell, and its later use in the classification of the 23 types of arches distributed in the building.

These works shed light on the subject, providing high precision tools based on techniques of point scanning, three-dimensional models, analytical geometry procedures, numerical methods, computer science and statistics, which allowed to reach the most rigorous conclusions. According to its authors, the object was to create a mathematical determination protocol tending to find analytical equations of arches with application to the study of architectural heritage.

Taking as a first specific case the arches of the Palau Güell façade they deduced that the profile that best suits them is the Rankine curve, with 99.75% coincidence.[3] The second curve closer to the shape of the arches is the ellipse with 97.82%, much more than the catenary and the parabola, the two curves recurrently indicated by the bibliography.[4]

APPROACH TO THE ISSUE

The Palau Güell is a magnificent urban palace that embodied the transition of Gaudí to maturity as an architect, constituting one of his masterpieces despite being still very young at the time of projecting and building it.

The recurring assertions that the twin arches of the main façade of the Palau Güell have catenary or parabolic profile are surely based on a certain similarity that at first glance they present with other arches that Gaudí used in this one and other works which do respond to the profiles mentioned, although it must be said that they are generally classified with a certain lightness.

The arches of the facade of the Palau Güell (Photo Fundación Antonio Gaudí)

This confirmation is quite interesting because the Rankine curves require of complex mathematical operations for their determination. The purpose of this paper is to outline a hypothesis about the procedure that Gaudí could have followed for the use of this figure difficult to draw, based on the knowledge and means available to him, and to investigate the motivations that led him to make use of such profile.

Nobody mentioned until now the Rankine curves, probably because they do not appear in the treaties of elementary geometry because unlike the aforementioned these are not traceable with the usual geometric methods.

Their name comes from William Rankine (1820-1872), a Scottish engineer and physicist among the pioneers of thermodynamics. It is known as the "Rankine curve" the one that describes the cycle of conversión of heat in work, the basis of the process of thermoelectric power plants. His professional work led him to develop also a method of tracing curves applicable to railway networks.[5]

Rankine was author of treatises on applied physics that included various branches of this science. His numerous studies also comprised the field of construction and especially that of applied mechanics.[6] In a text that we will cite later Rankine explained the properties of an arc subject to variable vertical loads, which he called "stereostactic arc".

Why Gaudí wanted to provide the arches of Palau Güell a shape similar to some simple outline curves, such as conics or catenary, but instead requiring complex operations for its delineation is a matter to be clarified. The same about the lack of correspondence of that curve with the one dictated by the state of loads of the facade, as we will see in the development of this paper.

It must be said that Gaudí was always open to using techniques alien to architecture, incorporating them intelligently. As an example when he commissioned a naval workshop to bend the laminated profiles that support the wavy façade of Casa Milà.[7] Or when using in an unprecedented way photography as a design tool for the church of Colonia Güell.[8]

And as for the economic resources set at his disposal in the construction of this palace, it is known they were large as reflected in the well-known anecdote that involved the secretary of Eusebio Güell.

The project and construction of the Palau Güell was intense and rich in vicissitudes. Several changes were made on the fly, affecting even structural elements to favor functionality.[9] The elaborate definition of the façade is widely known, requiring apparently up to 28 different solutions.[10] This only makes clear that the form of the portals was not something that arose at random but as the result of a process in which factors of diverse kind intervened: mechanical, constructive, functional and symbolic.

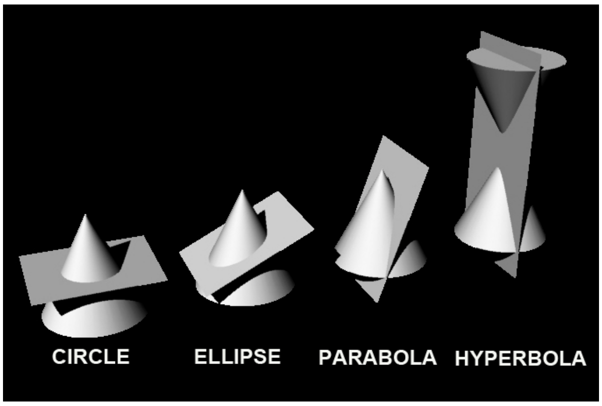
As an avid reader of Viollet-le-Duc Gaudí must have taken note of what the master recommended in his "Dictionnaire" about refraining from solving equilibrium issues by algebraic formulas that "are usually useless for the practical".[11]

SOME GEOMETRICAL AND MECHANICAL CONCEPTS ON ARCHES

For the development of this study, it is convenient to review some concepts about the layout and properties of some arc profiles. Geometrically an arc is a continuous curve that joins two points. In architecture the concept reaches another dimension given its status as a structural element. The most historically used profile for tracing arches was the circular one, which has an explanation in the simplicity of its construction, although as we know and we will expand later, it is not generally the form dictated by statics.

The circle is part of the family of conics, figures that can be drawn from plane sections of a cone. In this case it is a section perpendicular to the axis. The fundamental geometric condition of the circle is that all its points are equidistant from a center or focus. Ellipses, other conical curves, are plane sections at infinite angles ranging from 90 ° to the angle between the generatrix of the cone and its axis.

They have two focus with which all the points of the curve have the following relationship: the sum of their distances to them is the same. A circle can be said to be an extreme case of an ellipse in which the two focus are conjugated.



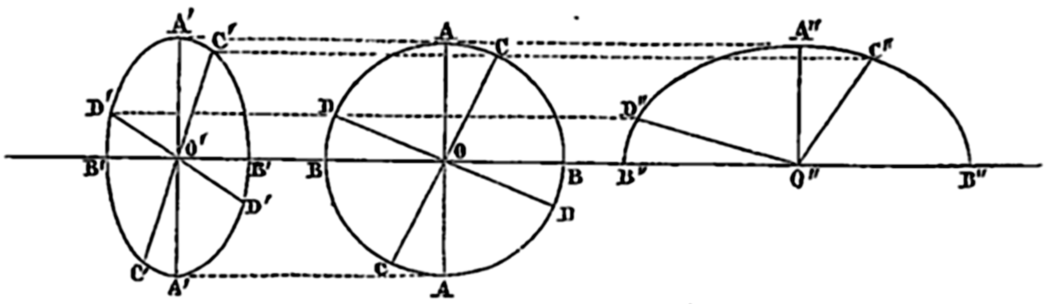
Generation of conic curves

The circular arc is the equilibrium curve against the pressures produced by a uniform fluid. The normal pressure at each point of the arc is equivalent to a pair of conjugate pressures of the same intensity in perpendicular directions, and the tangential force at one point of the arc is the product of the normal pressure multiplied by the radius at that point.[12] As the radius is the same at all points, the force along the curve is always the same too.

If we establish an orthogonal coordinate system and all the pressures in one of the directions are multiplied by a constant value, the two pressures at each point will be different from each other, and the arc intended to support that pressure system will be the parallel projection of a circle, that is, an ellipse, where the radius is variable and so are the tangential forces along the curve. The operating pressure scheme is no longer that of a uniform fluid.

As one of these pressures becomes infinitely less compared to the other, we finally arrive to the other extreme case of ellipse: the parabola, the conic drawn by a plane that cuts a cone at the angle that the generatrix of the cone forms with its axis. One of the focus of the parabola is in finite space while the other is in infinity. The points of the parabola are equidistant from the focus and from a line perpendicular to the axis of the curve that we call the directrix.

The acting pressure scheme now resembles the usual one in architecture, essentially dictated by the force of gravity materialized by the pressure parallel to the axis of the curve, which in the the case of an arc is the vertical component. Thus, the parabola is the balanced form of an arc subjected to a state of vertical evenly distributed loads in the horizontal projection.

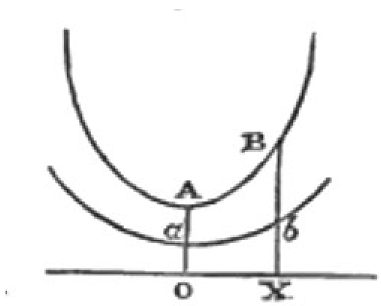


Circle and ellipse. From RANKINE, WILLIAM JOHN MACQUORN. (1876). P. 201

Finally, the conics include the hyperbolas, curves of two branches drawn by the sections produced by planes at infinite angles between the axis and the generatrix of a double cone.

THE CATENARY

Another recurring figure in the Gaudí bibliography, the catenary, is the inversion of the curve formed by the weight of a rope hanging from two points. A catenary guideline arc responds to an equilibrium state of increasing vertical forces in the horizontal projection from the key to the supports. This happens because each point of the arc has to bear an identical weight. As we move away from the key, each successive section of the horizontal projection supports more weight because it includes more points on the curve. This increase must respond to precise values to form a catenary, which can be calculated analytically and graphically. An easy way to do this is by plotting a horizontal line on the key and measuring at regular intervals specific distances from that line that determine each point on the curve.

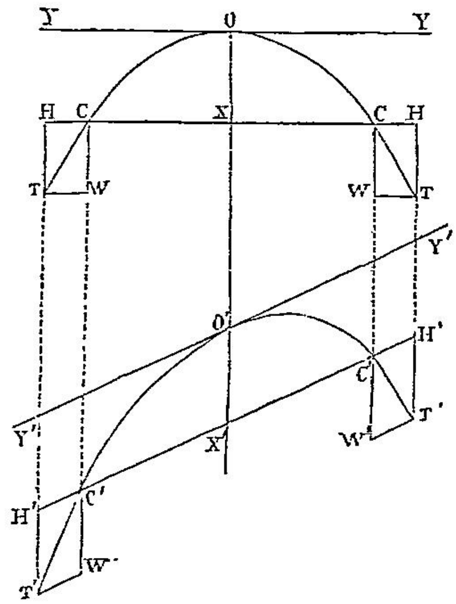


The catenary and the transformed catenary. From RANKINE, WILLIAM JOHN MACQUORN. (1862). P.

William Rankine, who gives his name to the curve adopted by the arches of the Palau Güell façade, mentions in his treatises the figure he calls "transformed catenary", a curve drawn from a catenary preserving the horizontal coordinates from the axis, but altering the verticals in a certain percentage. The resulting figure is another catenary drawn on a different scale where the load in each span of the arc is proportional to the area between that arc section and the a horizontal reference line ("extrados" as we will see Rankine defined it), and the intensity at each point is proportional to the ordinate of that point, two properties of the catenary.

THE STREOSTATIC ARC

The state of charges capable of defining a pressure curve adjusted to a regular figure easily drawn (circle, ellipse, conic or catenary) is not frequent. In general the incidence of loads tends to separate it from those regular paths.



The estereostatic arc. From RANKINE, WILLIAM JOHN MACQUORN. (1876). P. 219

Certain rating of variation of vertical and horizontal forces on an arc generate specific curves: We come here to a particular type of arc that the Scottish scientist describes in his bibliography: the stereostatic arc.[13]

These are arches destined to support in any point that two conjugate pressures: A vertical pressure V, variable from the key up to the supports, and another horizontal pressure H (it could be inclined too), constant, located in a same plane. One condition established by Rankuine is that the vertical loads must be distributed symmetrically with respect to the axis of the arc. A profile of this type is that described by the arches of the portals of the Palau Güell.

The mathematical determination of the stereostatic arc arises from hyperbolic cosines in combination with exponential functions and requires mathematical operations that are very complicated for the needs of the architects. However, these arches can be drawn on the basis of a hanging model, placing increasing weights from the key to the supports, representing the acting loads in each section.

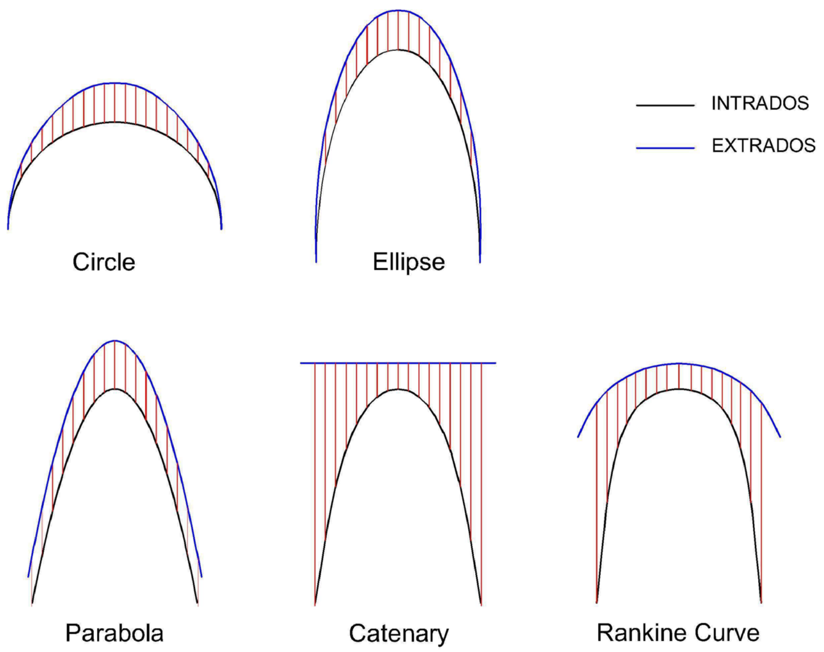
LINEAR ARC UNDER VERTICAL LOAD

To study the mechanical behavior of arches, we turned again to Rankine, who defined "linear arches" as those formed by an inverted and stiffened rope capable of preserving its shape and of resisting compression, thus representing a balanced arc under the forces acting on it.[14]

Rankine pointed out the concepts of "intrados", the figure of the arc itself, and "extrados", the line that joins the upper ends of the verticals drawn upwards from points of the intrados with lengths that represent the intensities of the loads. The curves we have described above have certain characteristics in relation to these definitions.

Thus, ellipses and circles, closed curves that balance equal normal pressures throughout their development, if subjected to only vertical forces such as gravity they adapt in the key to the vertical normal pressure while at the points where the curves are cut by their horizontal axis maintain equilibrium if the vertical component is zero, while the horizontal is maximum, equal to normal pressure. The extrados is thus an elongated curve in the vertical direction. Such state of loads, decreasing from the key to the supports, is infrequent.

The parabola, as an open curve (closes at infinity) is as we said the curve that balances uniformly distributed vertical loads. In order for the weight of the spandrels not to affect



Intrados and extrados following Rankine's definitions this scheme, there must be a considerable load coming from the upper structure. The extrados is another vertically displaced parabola.

When the distribution of the charges is such that their intensities are increasing symmetrically from the key in horizontal projection, the equilibrium curves are stereostatic, that is "Rankine curves".

A particular case of this type of curve is the catenary. We have already said that it can be plotted by measuring certain distances from specific points on a normal horizontal axis. Therefore the extrados of a catenary is a straight line perpendicular to the axis.[15] For the same reason, it is the arc that best adapts to a construction scheme with solid spandrels that influence the top load.

THE STATE OF THE STATIC CALCULATION OF ARCHES IN TIMES OF THE CONSTRUCTION OF PALAU GÜELL

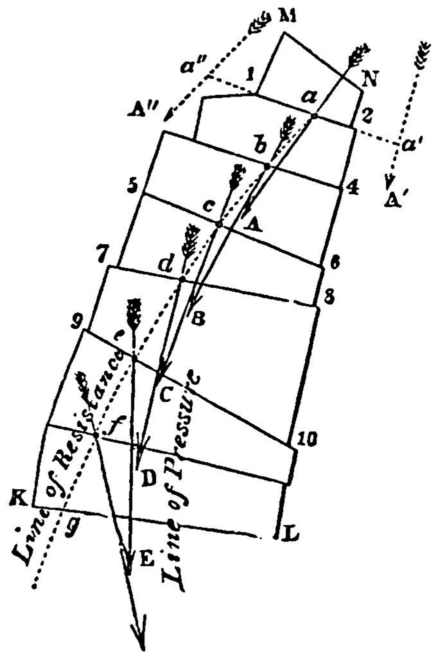
At some point in history, the lintel sustained by two vertical elements was replaced by two inclined linear elements mutually supported by opposing forces.

From there to the fundamental idea of the arc there was only one step: When adding a third intermediate element. There began the action of the rubbing and the interacting forces between them and as responsible for the equilibrium. The voussoir arc is the evolution of that primary idea, and its essential basics remained the same.

To ensure that these efforts do not affect stability, certain conditions between the height and the span of the arc must be met. The use of a certain profile is not a guarantee of equilibrium.

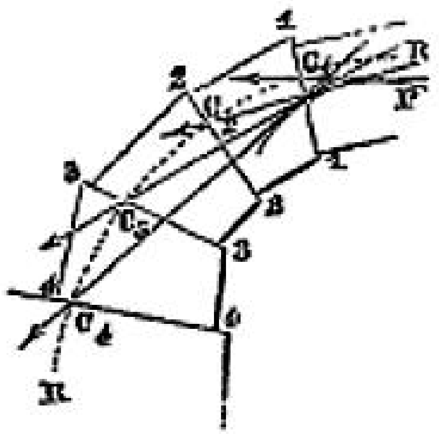
The problem was historically solved with the addition of loads to verticalize the results. In the absence of significant thrust, it was enough with the weight of the spandrels. The vaults were equipped with buttresses whose evolution were the flying buttresses. The same happened with the transverse arches and ribbed vaults. Everything until the nineteenth century was based on experience and intuition, without any technical support.

Along the century evolved the concept of the "curve of pressures". The methods of "Graphic Static", so popular when Gaudí was studying at the School of Architecture of Barcelona, allowed architects to have a simple tool to design structures. The voussoir arches, essential elements of the construction and components with great protagonism in the architecture of 19th century, required then, before the irruption of the reinforced concrete, that their resistant pieces contained to the curve of pressures.



Line of resistance and line of pressure.
From MOSELEY, HENRY. (1843). P. 380

Henry Moseley stated in 1843 that the equilibrium theory of the equilibrium of a structure is established by the line that determines the point of application of the resultant of the pressures on the contact surfaces ("line of resistance") and the line that defines the direction of that resultant ("line of pressure").[16] The equilibrium conditions of the system are satisfied when these lines do not exceed the limits of the arc and when the second one does not affect the joints at an angle greater than the limit angle of resistance.[17]



The Curve of pressures. From RANKINE, WILLIAM JOHN MACQUORN. (1876). P. 256

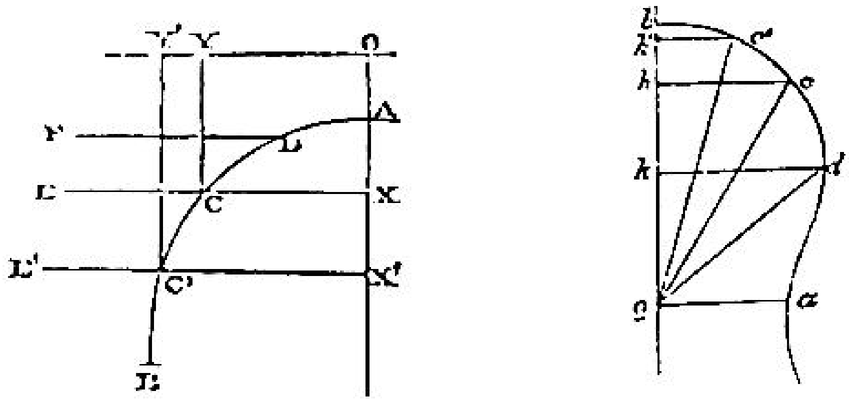
William Rankine established the curve of pressures (also called "curve of equilibrium"), as the curve inscribed in the polygon formed by the lines that pass through all the centers of resistance (centers of pressure in the joints), as an application of the "linear arches".[18]

He also defined it as a succession of tangents to a polygonal whose pieces and joints are so numerous that its figure can be considered as a continuous curve that constitutes the required balance curve.[19] Summarizing the conclusions of contemporary studies Rankine stated that the stability of an arc is safe if a linear arc, balanced under the forces acting on the real arc, can be drawn in the central third of the joints.[20] An essential condition is that all forces that maintain the equilibrium of the arc must be pressures acting from the outside in.[21]

Numerous methods were developed for drawing the curve of pressures, which generally consisted in establishing two points of passage of the curve in each half-arc: in the keystone, the point of application of the horizontal thrust, and the "point of rupture", in the intrados, located at an angle of 60º with respect to the axis of the arc, which is the point where the curve of pressure meets the the intrados.[22] Trhrough the latter passes the result of the portion of the arc included, with direction tangent to the curve.

The bibliography established that among all the possible lines the one that passes through the upper limit of the middle third in the crown, and by the inner edge of the middle third in the rupture joint is the one that exerts the least amount of thrust. The various methods adopted this practical rule. In this way it could be composed a triangle that would serve as a basis to draw the polygon of pressures with the sum of the vertical loads of each section of the arc forming the third side that represents the weight of the entire portion considered applied at its center of gravity.

Rankine explains the rationale for the method: in a scheme of distributed vertical loads in any way it is possible to determine a system of conjugate pressures that establish equilibrium. They can be graphically solved by constructing an auxiliary polygon to which the tangent at a given point (C) is translated, which represents the compression along the arc, and the sum of the loads supported by the arc AC (Oh). Closes the triangle the segment hc that represents the horizontal component.



Method for determining equilibrium pressures. From RANKINE, WILLIAM JOHN MACQUORN. (1862). P. 213

In summary, in Gaudí's times it was set that the stability of the arc was guaranteed if the pressure curve was contained in the arc itself, when its incidence in each joint was close to the perpendicular and if the thrusts in each point of the arc and in its supports (the horizontal component of the resultant) could be absorbed by some part of the construction, be it the arc itself or some extra element.

THE GAUDINIAN ARCHES

The interest of Gaudí for the equilibrated arches avoiding aesthetic prejudices was manifested since his early works.[23] Even before projecting this palace he had experimented with the catenary arc at the Cooperativa Mataronesa, the Finca Güell and at the disappeared waterfall-viewpoint of the Casa Vicens.[24] In the last two, he built monolithic arches, experimenting with the catalan vault. This technique will evolve until reaching the development of vaults in the form of warped ruled surfaces in many works.

At the time of the Palau Güell Gaudí he used parabolic brick arches in the Teresian College.

It is known his knowledge of the graphic methods of structural design and of the diverse profiles of arches according to the state of loads. Gaudí employed the graphic methods in use at the end of the 19th century, which gave reliability. In that sense, he was immersed in the spirit of the time that pretended to know a possible state of equilibrium of a structure.[25] He left registry about this in his youth writings.[26]

The graphic method in a simplified version by Gaudí himself was his tool for the determination of the stability of two-

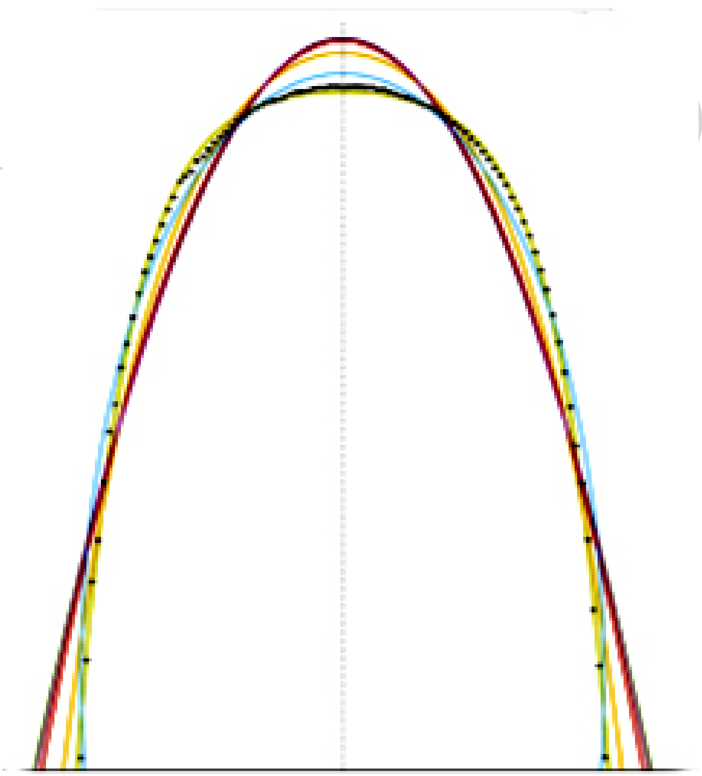
Arches of the stables of Finca Güell (Photo Fundación Antonio Gaudí) dimensional elements.[27] In many cases the arches fit to the curve of pressures thus obtained, prioritizing mechanical efficiency over the aesthetic preconceptions. In this he differed from the generality of his colleagues who used this method to verify that the previously established form complied with the equilibrium conditions, making sure that this curve was contained in the designed figure.

In addition to graphic methods, Gaudí used hanging models as a design tool. Although there is no documentation on the materialization of this type of models until at least the first studies for an unbuilt chapel in the Park Güell around 1900, there is no reason to suppose that they couldn't have been used before, certainly not with the complexity developed in that three-dimensional project and much less that in the later church of Colonia Güell, it is plausible that he has done so in two-dimensional studies.
A few later, approximately in 1908, for the construction of the arches for the attic of the Casa Milà he drew the curves with hanging chains that formed catenaries to which he applied scale weights that materialized the influence of the spandrels and slabs, as shown at the photographs of the work.[28]

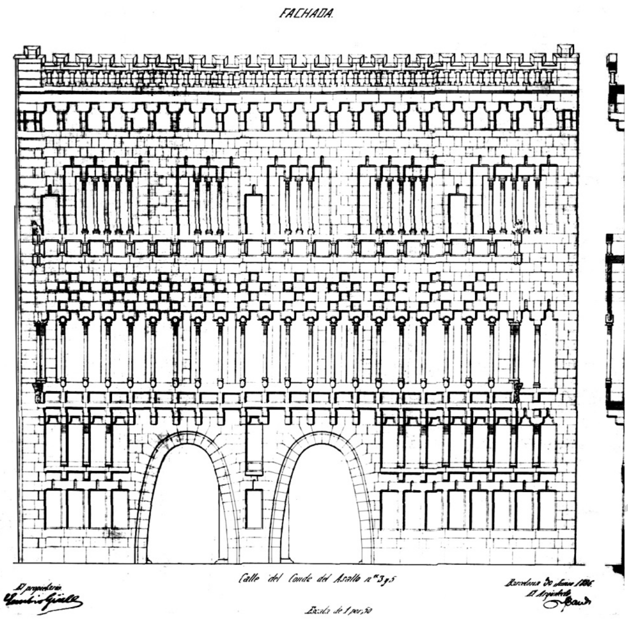
Given the knowledge and the experiences he already had in his youth, Gaudí could have used this technique in tracing the arches of the façade of Palau Güell, a procedure he should know through the books that reflected some experiments in this field developed since two centuries ago that he assiduously consulted as we know by his biographers. One of the texts that Gaudí surely consulted in his readings in the library of the School of Architecture was the well-known treatise of Jean-Baptiste Rondelet.[29]
Gaudí used these methods knowing that the inaccuracies they may entail are of little significance in the field of construction. This way he proceeded to make the templates that were transferred to the stone or brick.

THE DEFINITION OF THE SHAPE OF THE ARCHES OF THE FACADE OF THE PALAU GÜELL

As already mentioned, the arches on the façade of Palau Güell conform to a Rankine curve with 99.75% coincidence and to an ellipse with 97.82%. It would be attractive to assume on the basis of this apparently small difference an attempt by Gaudí to draw an ellipse with a simple geometric method instead of the complex Rankine curve. This dissimilarity of 1.93%, represents nevertheless in practice a considerable value even accepting that all construction has margins of



Comparison between Rankine curve, ellipse, parabola, catenary, hyperbola and hyperbolic cosine applied to the arches of the Palau Güell facade. From SAMPER, ALBERT; GONZÁLEZ, GENARO; HERRERA, BLAS (2016) tolerance: For 5.90 m. height of the arches the discrepancy is on average 11.4 cm. and it reaches around 16 cm. in some points as shown in the drawing that accompanies the study of Samper, González and Herrera. There we can see how the ellipse moves away upward in the key while in the rest of the arc it intersects with the Rankine curve.



Facade of the Project signed on july 30, 1886

It can be checked graphically that the arches shown in the façade of the project signed on June 30, 1886 have the same shape as those finally built, despite there being some changes such as the absence of the two tribunes on the second floor or the shape of the crowning. This layout was drawn in 1:50 scale, which would mean errors of more than 3 millimeters for an arc about 12 centimeters high. Furthermore, if an ellipse had been attempted to be drawn, it is very likely that the well-known method of a wire fixed in the two foci would have been used, which leaves no room for error. From our perspective, inaccuracies of this magnitude are unlikely in someone like Gaudí, punctilious and involved to the detail in every aspect of his works.

According to what was said before, the following can be highlighted as objective data:

1st) The arches of the façade of the Palau Güell have a very tight guideline to an Rankine arc, a fact that arises from the studies mentioned at the beginning that state the question with a scientific basis.

Rankine curves are very complicated figures that require complex mathematical calculations, though they can be drawn by hanging models,.

2nd) Graphic methods were commonly used among architects as a means of verifying the stability of a structure, as explained above and we will not repeat so as not to overflow.

3º) In Gaudí's time, the use of hanging models as a project tool was documented through a diverse bibliography.

On the basis of this knowledge a hypothesis will be attempted about the way in which the arches of the Palau Güell façade were projected, the intervening factors and the motivations that defined its guideline.

The process will begin with a study of the curve of pressures that arises from the analysis of the loads that act on the façade of the building according to the method that we documentally know Gaudí used.

With this graphic procedure it will be verified its adaptation to the guideline of the arc and the stability of this element.

The possibility of using a hanging model loaded with the same weights will be studied.

The shape of the arc finally adopted by Gaudí will be compared with the alternative design option that is known for having been published before the destruction of Gaudí's archives. [30]

Finally we will try a geometric determination of the shape of the arches based on graphic methods and we will outline deductions and conclusions from the developed analyzes and the results obtained throughout the study.

LAYOUT OF THE CURVE OF PRESSURES WITH THE GRAPHIC METHOD

Now we'll applicate to the arches of the Palau Güell façade the graphic method used by Gaudí for the drawing of the curve of pressures, consisting of the division of the arc in vertical sections, determination of the direction of the resultant in the point of rupture, drawing of the triangle of forces and composition of the forces on the arc.[31]

As a first stage the loads that intervene will be determined. To do this the plans and the various studies carried out by the professionals who took part in the restoration works of the building will be taken as a basis and source of information.

FACADE:
It is a bearing facade of limestone coming from the quarries that Mr. Güell owned in Garraf, at the south of Barcelona, about 70 cm. thickness, although variable along its vertical development due to the presence of numerous windows placed in an asymmetric scheme.

Contrary to the academic canons, the ground floor portals are not located in the center, although they compose a scheme of axial symmetry with other elements distributed in the upper floors where the arches are the protagonists. The differences between the plan of the facade of the Project and the built one do not affect this analysis provided that the loads are almosst the same.

LOAD ANALYSIS:
The loads come from the roof, the third floor, the second floor and the main floor along the 23.50 meters wide of the facade. There Gaudí placed a great tribune where a succession of columns in two rows regulates the distribution of the loads along the entire wall above the keystones of the arches.

Over the ground floor there is a mezzanine traversed by the access and exit streets of carriages (the height of these streets reaches 6.50 meters), part of which is supported by beams situated perpendicular to the plane of the facade that discharge into the wall at 3,80 and 4.30 meters high. Four of these beams directly influence the arches in a way that will be analyzed later. The 4 slabs that load on the main facade are, from top to bottom:

ROOF:
According to the description made by the restorers of the building based on tests, the construction system of the roof slab is as follows:

Metallic beams and ceramic vault, with spandrels filled with rubble and lime mortar, over which extends a board formed by four common brick leaves of 29 x 14 x 2 cm., The first sheet stuck, glued with plaster, the two top sheets placed diagonally in zig-zag and finally the tiled covering.[32]

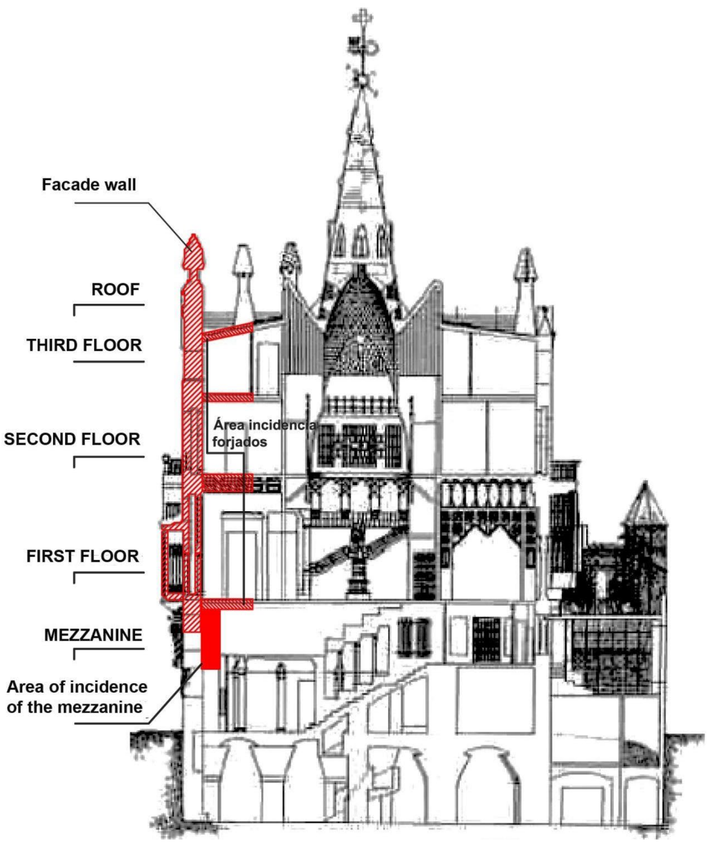
THIRD FLOOR:
Located under the roof, whith the service rooms, is a wooden structure formed by large beams of 40 x 30 cm. in one direction, and in the other 30 x 30 cm, rotated 45 degrees that supports a sequence of various layers of mortar and ceramic pieces with 15 cm. thickness.[33] This configuration responds to

the need to obtain the maximum acoustic isolation, as demonstrated by a study carried out for this purpose.[34]

SECOND FLOOR:
This is an original wooden supporting structure that combines the sustaining and the formal function as it can be seen as a coffered ceiling covering the halls of the first floor.[35]

FIRST FLOOR:
This structure is formed with beams made of metal profiles, as described by the restorers in the restoration reports.

As it was said above, 4 beams receiving the mezzanine loads are supported by the facade in the area of the entrance arches.
Due to functional reasons these beams unload at a different height: 4.30 meters the central and 3.80 the lateral ones, situating their axes to a distance of 1.65 m. on both sides of the axes of the arches. The arches forming the access and exit doors of the palace combine functionality and static. It is inscribed in each of them a vain for the passage of carriages with 3.60 m. height and 3.00 m. width. The shape of the arches had to adapt to this functional requirement.



Cross section with study of the loads that influence the arches

MEZZANINE:

The arches of the facade of the Palau Güell. Hypothesis about its conformation

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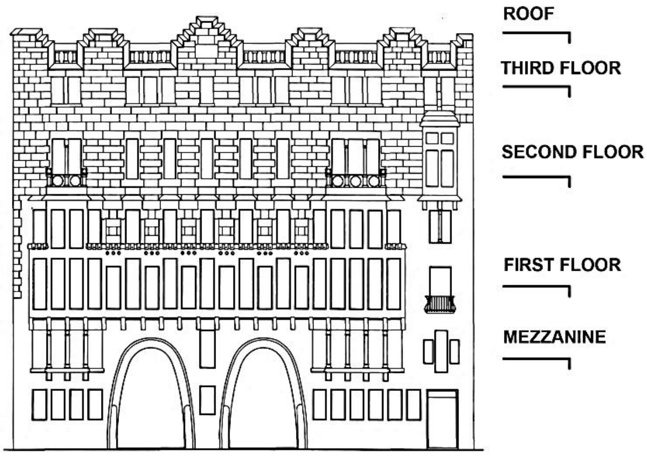
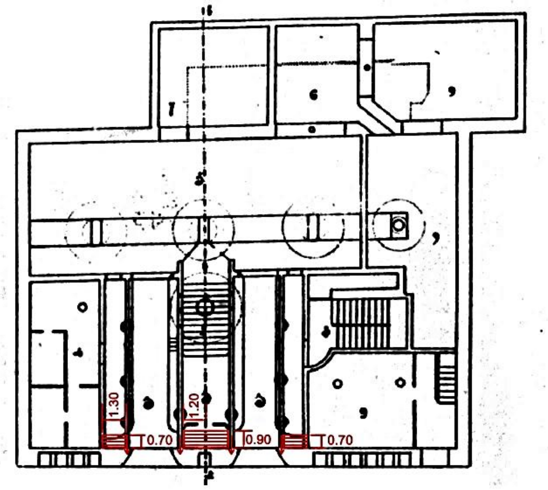
FACADE



LOADS MEZZANINE



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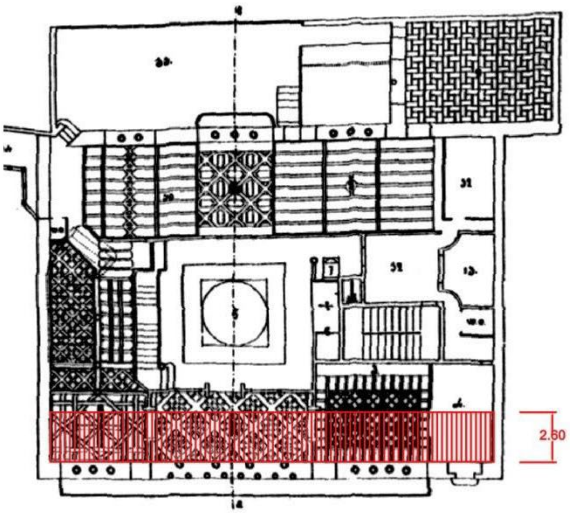

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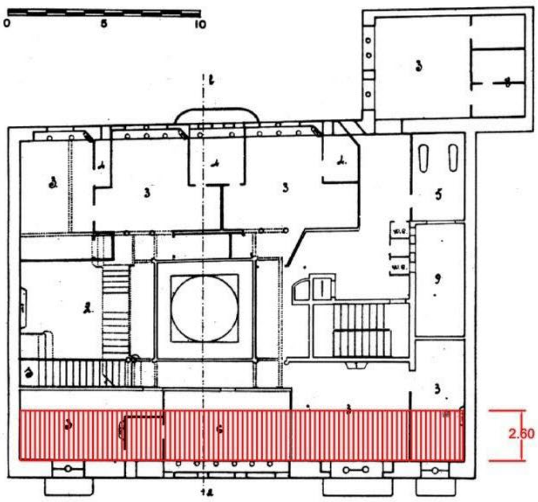

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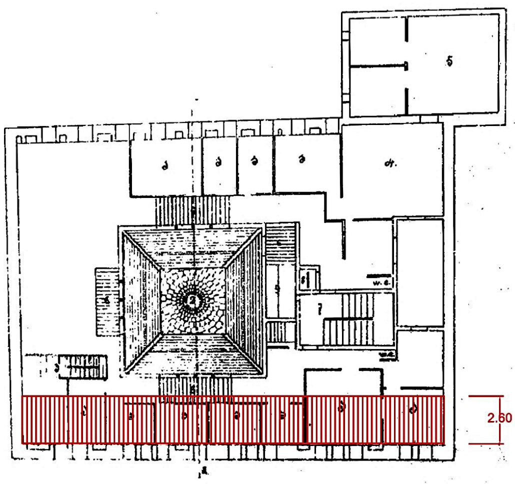

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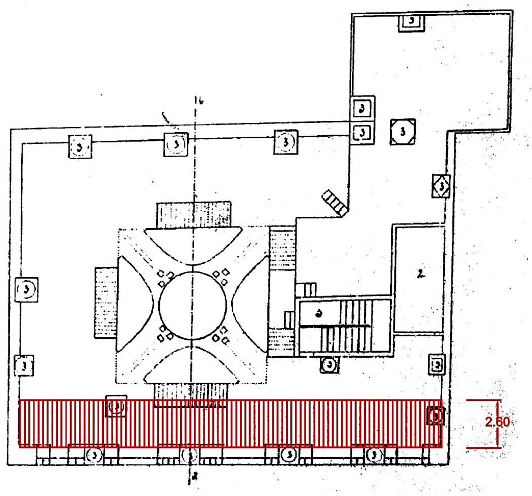
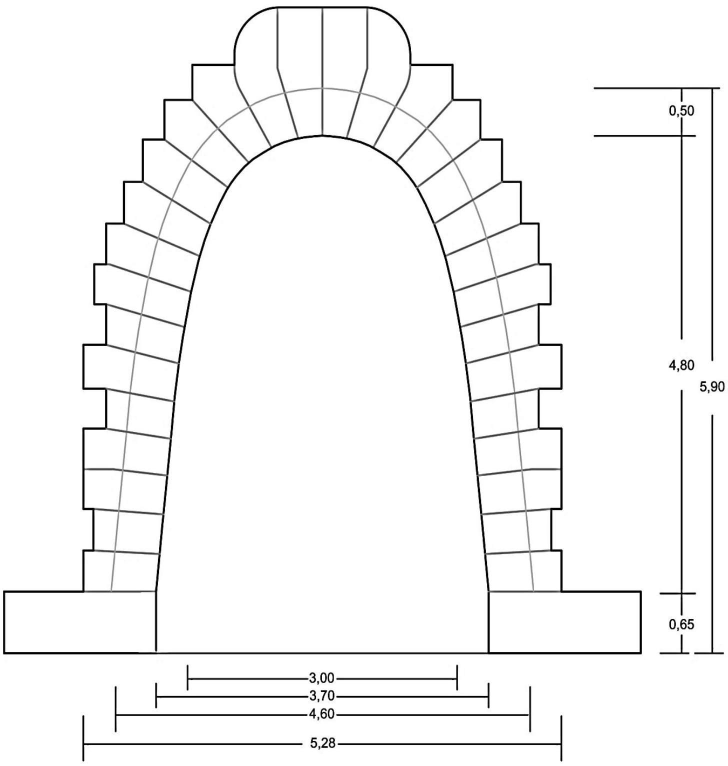


Facade and Plants with study of the loads that influence the arches

Based on this information, the load analysis was carried out, which is attached in Annex I, from which the following results emerged:

TOTAL DISTRIBUTED LOAD OF THE WALL UNDER GROUND FLOOR DECK (Decks + wall facade):
22.460 kg/m.
MEZZANINE BEAMS LOADS:
Lateral beams: 1.675 kg.
Central beams: 1.867 kg.



Arc of the facade of the Palau Güell. Voussoirs and measures.

LAYOUT OF THE CURVE OF PRESSURES

To perform the plotting of the pressure curve following the method used by Gaudí, the calculated loads distributed in sections of uniform length were applied in one half of the arch (12 sections of 0.20 m.). The point of rupture was located at the joint with an angle of 60º from the axis and the horizontal thrust on the crown was located at the outer limit of the central third.

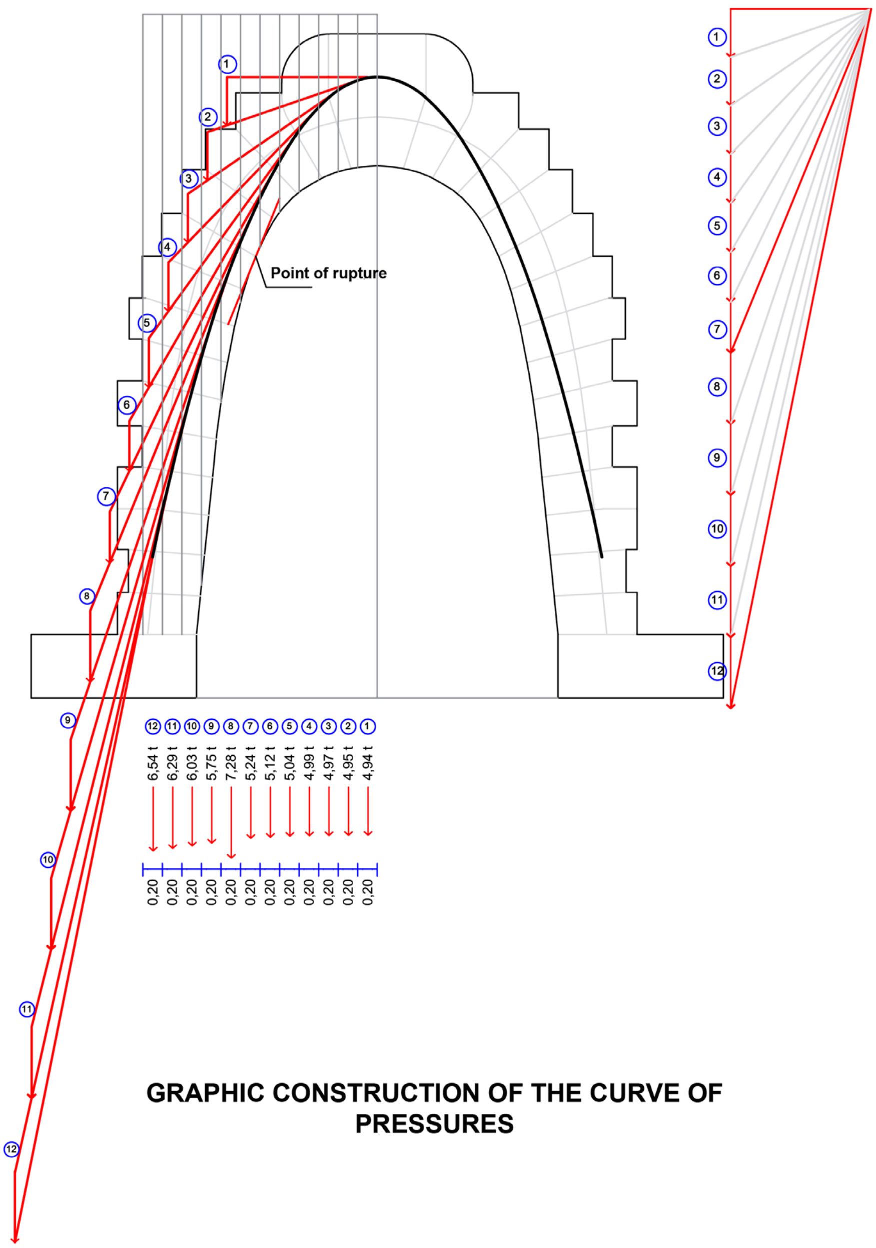
Once the forces were composed, the results are as follows:

The curve should be in its first section near the crown quite similar to a parabola as a result of which the charges are very alike in those sections. However, it is markedly modified by the influence of the loads of the mezzanine beams that affect section 8 and alter that curve deforming it inwardly.

There acts besides the progressive increase of the load in each section as a product of the geometry of the arc itself. The equilibrium is ensured since the curve of pressures is contained entirely within the arc between the keystone and the point of rupture. In this section of the arc the path is that recommended by the bibliography: in the inner limit of the central third.

As well as we approach the supports the resultant tends to move away from the central third. This does not mean a danger to stability since in that area the arc is embeded with the wall because of the interlocking of the voussoirs. The vertical load and the wall counteract the thrust and the set results balanced.

As an evident conclusion it is observed that the curve of pressures does not resemble the intrados form of the arc. This lack of correspondence, although doesn't imply consequences for the stability of the structure, is quite striking given the use Gaudí usually made of unconventional arches in



GRAPHIC CONSTRUCTION OF THE CURVE OF PRESSURES

Graphic construction of the curve of pressures
Drawn following themethod that Gaudí used

architecture, adjusting them to the dictates of equilibrium.

In this case another kind of motivation seems to have prevailed related more to functional and formal-representative issues that will be later analyzed.

One theme to be highlighted is the voussoir scheme designed by Gaudí for the arches. The enormous pieces in the area of the keystone of 1.35 meters high whose function is to raise the center of gravity where the load is applied are striking.

It is also interesting that the keystone is in an unusual way divided into two pieces by matching a joint with the axis of the arch. It is also remarkable the shape of the voussoirs of the keystone with rounded profile generating a link with the ashlars of the wall.

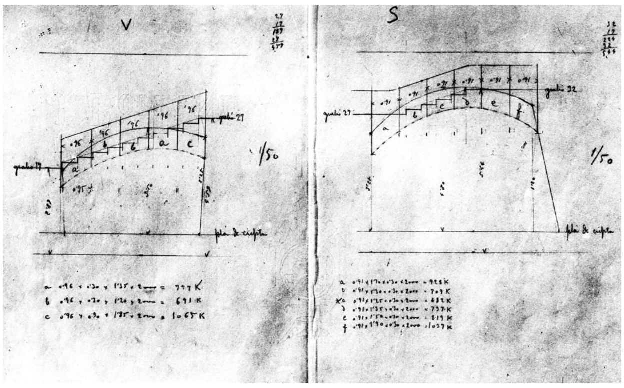
Finally, attention should be paid to the vertical sections that raise the start of the arches in 0.65 m. on the ground level, something not repeated by Gaudí in other works where the arches start inclined from the ground.

All these observations add more interest to the formulation of hypotheses about the possible causes that influenced the shape of the arches.

PROBABILITY OF THE USE OF A HANGING MODEL

The possibility that Gaudí used a hanging model to trace the arches of the Palau Güell façade was evaluated by means of an experiment with a model to which weights in scale were incorporated representing the loads calculated for the points of the arc.

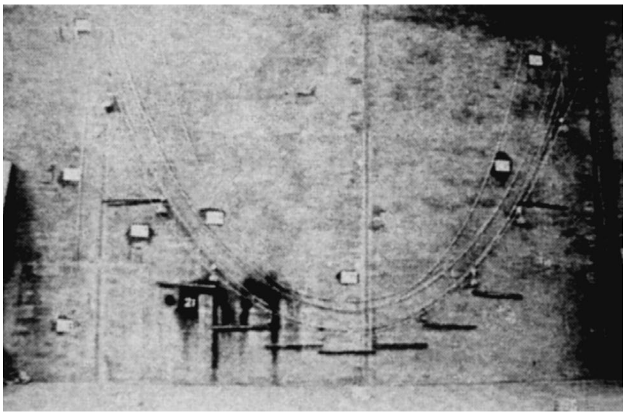
It was reproduced the method that is documented that Gaudí used in the project of the Colonia Güell church: study of the loads, transfer to scale of the weights to the hanging model and verification of the form obtained through photographs. This procedure is known by some preserved documents although it is necessary to point out that in that case it was a complex three-dimensional system.



Sheet with calculations for the hanging model of Colonia Güell church

For the arches of the attic of Casa Milà Gaudí used the same method. Although the narration of the builder Josep Bayó only refers to hanging chains without added weights, the preserved photograph reveals what appear to be small bags that would contain weights.

The result of this experiment was that the hanging model according to the load scheme that affects the arches of Palau Güell does not



Photograph taken by the builder Josep Bayó of one of the hanging models for the determination of the shape of the attic arches of Casa Milà

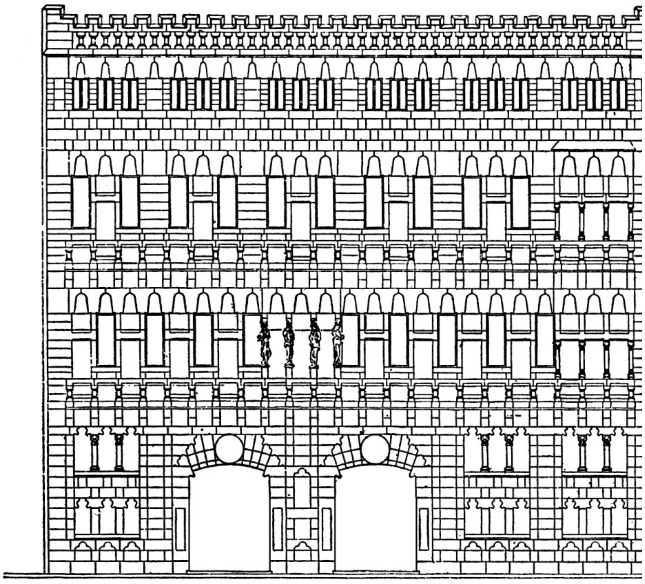
Photograph of the study model loaded with the loads of each section

adopt the profile of them but one very similar to that obtained by the graphic method.

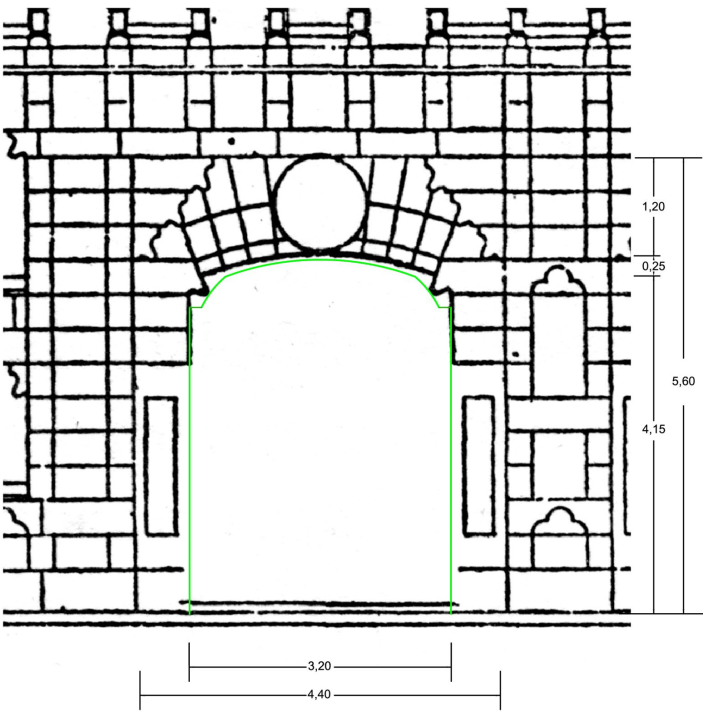
THE DISCARDED ALTERNATIVE PROJECT

It has already been said that thanks to the book of his biographer Josep F. Ràfols we know the drawing of one of the 28 facade studies carried out by Gaudí for the Palau Güell. An analysis of this fortunately preserved plan provides some interesting data for this study.
First it must be said that in general lines the façade reflected in the drawing does not differ much from the one finally built in terms of the arrangement of windows.

We can only repeat the differences in the galleries on the second floor and in the top of the facade, and emphasize the presence of four sculptures that represent human figures centered with the arches in the tribune of the First Floor, nonexistent in the constructed version. But this is a decorative issue that does not make the subject here treated.



Discarded alternative facade project published in RÀFOLS FONTANALS, JOSEP (1929)



Arc of the discarded alternative project

The fundamental difference from the geometric and static perspective is in the form of the entrance arches that in this case adopt a very low profile supported at a height of 4.15 meters on vertical jambs, which curiously are not reflected in the drawing. That shows that this part of the facade was in constant process of study and change. The arches have 3.20 meters span if we assume that the jambs are vertical.

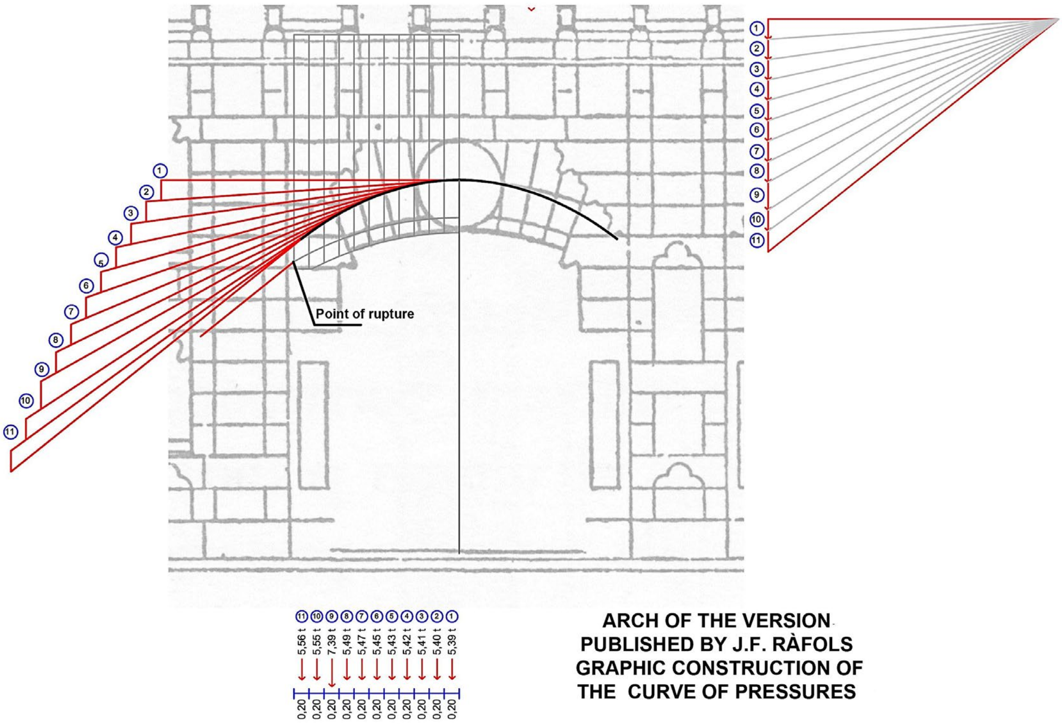
The voussoirs are pieces of regular width. As in the case finally built, although less pronounced, the voussoirs increase their height when approaching the keystone, what in this case seems to be a central piece of 1.20 meters height. There are two large circular medallions that seem to be the basis for the initials of Eusebio Güell that were built inside the tympans of the definitive arches.

We can see another way of interlocking of the voussoirs with the wall, in this case not in stepped form but with elaborate curves and countercurves. We must remember that the arches finally implemented have only rounded the upper part of the keystone voussoirs.

As an experiment we proceeded to draw on this arc the curve of pressures with Gaudí's graphic method too. As in the previous case, the half arc was divided into sections of 0.20 m. each, resulting 11 in this case. The calculation of the loads on the sections is developed in Annex II at the end of this paper.

The point of rupture was located at the beginning of the arc as formulated by the bibliography of the time when the arc has a development less than 120º.[36]

The shape of the resulting curve resembles a parabola because the load is very similar in all the sections except



Graphic construction using the method of Gaudí of the curve of pressures of the arches in the rejected alternative versión

in the one that receives the beams of the mezzanine, on the support.

In the last voussoir (support area) the curve escapes from the central third towards the outside, although this would not cause collapse when having the opposition of the walls of the façade that could absorb that thrust and balance the whole.

DEFINITION OF THE PROFILE OF THE ARCHES

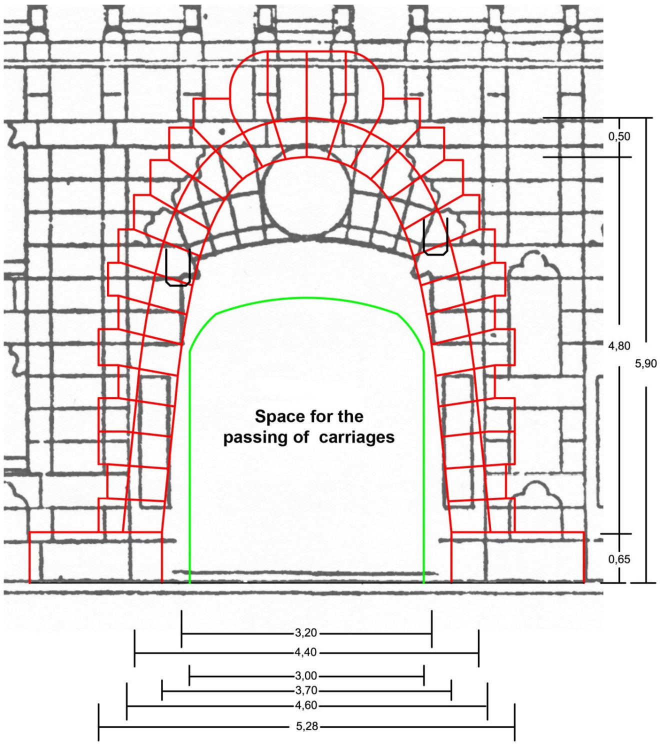
In the determination of the shape of the archs different conditioning factors affected besides the strictly static ones: We found functional factors, geometrical factors and symbolic and representative factors. Now we will analyze them:

FUNCTIONAL FACTORS

Gaudí gave dimension to the doors based on the functional need to allow the entry and exit of large carriages. Thus he defined a span of 3.20 m. wide and 3.75 m. height that rose in the center to approximately 4.35 meters, dimensions that fit comfortably in the project discarded with the low arches.

Apparently they had to be spacious measures for the functional needs because the profile definitively adopted in the construction lodges somewhat smaller doors, with a width of 3.00 m. and a height of 2.95 m. that in the central part reaches 3.65 meters below the tympan.

For this, Gaudí's ability to adapt resources to his interests had to come into play when he chose to raise by 65 cm. the starting points of the arches by incorporating small vertical sections. Through this resource was solved the problem of the passage of the carriages



The vain for the passage of carriages in the built arc and the discarded version superimposed

since all the necessary space was thus inscribed with comfort within the arc.

These seem to be the unique functional conditions that intervened in the design of the profile of the arches.

GEOMETRICAL FACTORS

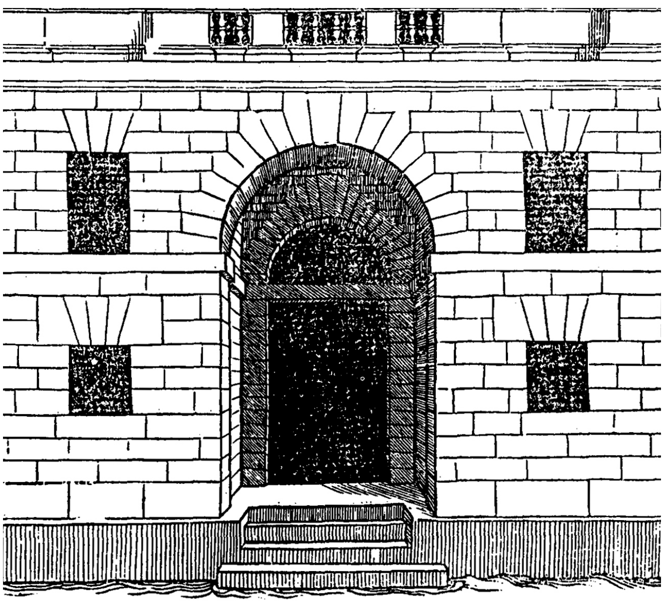
We have already talked about some relationships between regular curves and certain load states. It was also concluded that the most appropriate guideline from the point of view of stability in the case at hand is a curve quite similar to a parabola near the keystone and then extending outward as it approaches the springings.

If we compare this curve with the intrados of the real curve by drawing a parabola through the key and the springings, the difference is evident: the arc designed by Gaudí is much more open than the parábola, and to the ellipse and the catenary too. It is, as pointed out by Samper, González and Herrera, a Rankine curve, a "stereotactic curve" that results from the application of variable vertical forces and constant horizontal forces at the points of the curve.

This leads us directly to analyze the factors of symbolic-representative type, which will allow us to formulate the hypothesis about the unusual shape of these arches.

SIMBOLIC AND REPRESENTATIVE FACTORS

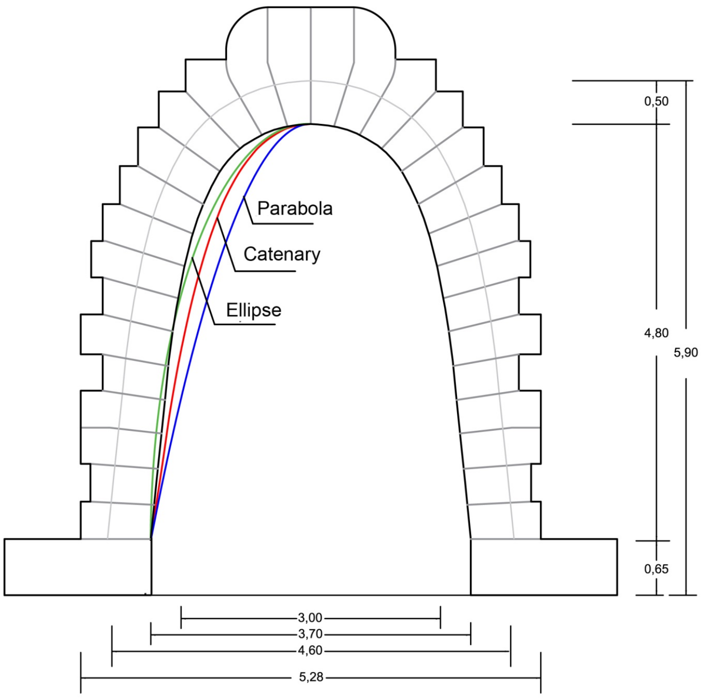
Eusebio Güell was determined that his Palace was a paradigm of modernity. So it was not accidental the coincidence of its construction with the most important international event that Barcelona lived until then, the Universal Exposition of 1888. Güell wanted to amaze the world with an exceptional palace and he did it. He cared to publicize it in national and international magazines, and the house was visited by kings and presidents.[37]



Section of the drawing of a palace. The text says: "Front madein the way that is customary in Venice".

It is difficult to think that so much dedication coming from an active and entrepreneurial person was not linked to an involvement in the design of the building and especially its public image, the main façade.

There are also many references to a relationship between Güell and Gaudí that went much further than a simple interaction between client and architect. They were friends and coincided in their vision of art - and architecture in particular - as a vehicle to communicate ideas. The Finca Güell, finished a short time before with its symbolism based on Greek mythology revised through the poem L'Atlàntida is a very clear example.



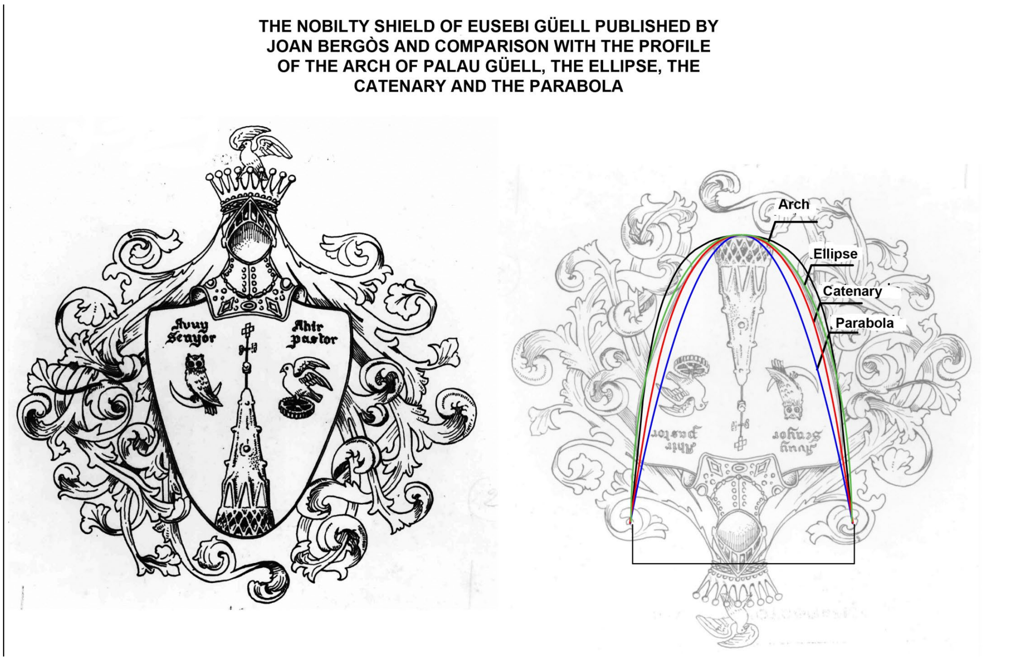
Comparison among the built arc and the ellipse, the catenary and the parabola

All this suggests that by common agreement Gaudí and Güell would have opted for the formally most fearless solution for the Palau Güell arches: resorting to non-traditional shapes. The catenary had been used in the stables and the lookout built in Pedralbes, a service building and a secondary construction. Now they would be the visible face of a great palace.

The shape of the arches must have been related to the interest in providing the entrance to the building with huge initials of its owner too. The very elaborate initials "E" and "G" of wrought iron that are placed in the tympans of the arches were surely too important for their owner to remain enclosed in circles on the entrance arches, as shown in the discarded alternative. They had to compose together with the spectacular shield located in the center an impressive group. The finally adopted arches with their great development in width at the top do open the necessary space.

The role of the voussoirs scheme in the stability of the arches has been analyzed before. It also has a representative function. It does not seem accidental that a palatial form has been adopted with clear inspiration in Serlio. In this context it should be noted that there are many parallels drawn between Eusebio Güell and the Renaissance lords.

The nobiliary vocation is reflected in the slogan inscribed on the coat of arms that Gaudí designed for Güell: "Avuy Senyor, Ahir pastor" ("Lord today, shepherd yesterday").[38] The shape of the emblem is striking: It is surrounded by a curve that contains, among other images, the Palau Güell dome. Bergós defined it as a catenary, but the curve in question is neither a catenary nor a parabola, nor does it coincide with the shape of the arc, but it is very similar to an ellipse, as shown by the superposition of these curves on the drawing.



The nobility shield of Eusebio Güell published by Joan Bergós and comparison with the profile of the arc of thePalau Güell, the ellipse, the catenary and the parabola.

A HYPOTHESIS ON THE DETERMINATION OF THE SHAPE OF THE ARCHES

Gaudí did not leave unattended details. In his projects everything was meticulously studied and redefined many times along the construction. This is confirmed by many testimonies and it is clearly observable when deepening the study of his works.

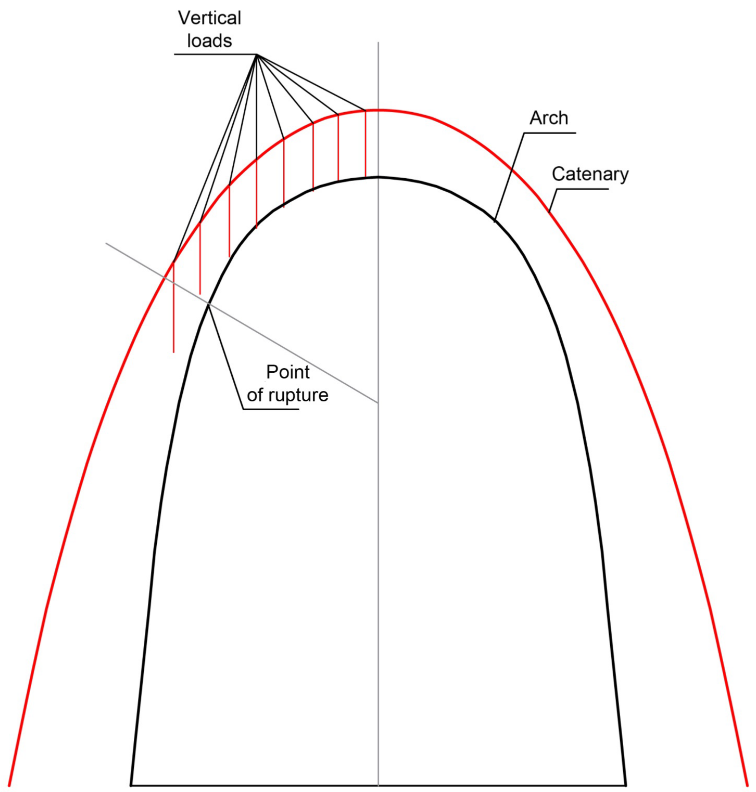
This leads us to think that a design issue as important as the shape of the arches in the portals of the Palau Güell facade should respond to some criterion. We have repeated throughout this text that it is not a catenary, a parabola or an ellipse and we have verified it graphically. We have shown that it does not respond to the curve of pressures dictated by the load state.

However the shape could not be any. It had to respond to some reason, to some law. Finally, a "stereotactic arc" or "Rankine curve" was adopted for the portals. This was demonstrated by the aforementioned investigations.

We have also analyzed the factors that influenced the determination of this unusual form. The importance of the symbolic nature of these elements is not minor. Its layout had to be linked in some way to an image of modernity. They had to synthesize art and science representing the spirit of the time.

Following this idea and as a hypothesis, a geometric study is included that seems to give an explanation of how could Gaudí have arrived to the definitive profile of the arches, or at least to a section of it. It should be clarified that it is a conjecture certainly impossible to be demonstrated.

We already talked about the relationships studied by W. Rankine between the shape of an arc -the *intrados*- and the line that he



Construction of the arc as intrados of an extrados with catenaric profile by applying the acting forces represented by vertical lines

methods that Gaudí had to know for having been published such as by Rondelet, as said above. From this curve graphically obtained considered as extrados, the acting forces already calculated can be applied in the indicated way and then their extremes draw the curve that would be the intrados.

This method isn't comparable to the transformed catenary, a curve drawn from a catenary preserving the horizontal coordinates of the points but altering the verticals at a certain constant value so that the condition that the intensity at each point is proportional to the ordinate drawn from that point keeps maintained, as explained before.

The concept instead is adjusted to the definition of stereotactic arches, the result of distributing variable vertical loads symmetrically with respect to the axis that passes through the key of the arc.

In any case, this procedure to draw the arc would explain the curve between the key and the point of rupture, but not its continuity until the springings. It is nothing but speculation, a simple observation of something that can be nothing else than a coincidence.

The same can be perhaps the great similarity of the profile of the arches with the Rankine curve. The reasons that led Gaudí to give such a form to the arches and the method used to trace them will remain as unknowns to elucidate. Perhaps in the unfortunate fire of

called *extrados* that arises by drawing vertical lines from the points of the arc that represent the intensities of the charges in each one, and then joining the extremes.
Following Rankine, if this procedure is applied to the arches of Palau Güell the curve obtained plots a catenary between the crown and the point of rupture. Beyond this point the line of extrados thus obtained moves away from the curve.

What is proposed here is that the curve of the arc would arise applying the procedure inversely: starting from a catenary as extrados the verticals can be drawn downwards representing the charges at each point. The new points thus obtained draw a curve that would be the final arc.

It is a procedure in which only graphic media are involved. A catenary is easily drawn with the workshop of the Holy Family in 1936 also burned the answers to these questions.

CONCLUSIONS

From all the above analyzed we will now draw some conclusions.

The shape of the arches of the portals of the Palau Güell seems to be an aesthetic and symbolic decision associated with the representative aspirations of the owner, Eusebio Güell, subordinating its correspondence to the directive marked by the static conditions.

The adopted arches meet the equilibrium conditions, as shown graphically, but do not follow the curve of pressures. How its guideline was drawn and translated to the stone is something that remains in conjecture.

Gaudí's way of building arches was in many cases designing them following the profile resulting from the loading conditions. That guaranteed stability, as he could learn from the bibliography of his time. This marked a difference with the majority of architects who used the graphic methods to verify the stability of the projected forms. In this case he departed from his rule, probably in agreement with his client and friend.

In the supposition that in order to achieve the shape best adapted to equilibrium a hanging model with weights representing the loads to which the arches are subjected was used here, in the same way Gaudí did not adopt that profile in the end. Later he would use the method of the model for the arches of the attic of La Pedrera and then he will develop a three-dimensional system for the design of an entire building: the church of Colonia Güell.

However, the stability issues were not alien to the design of the arches of the Palau Güell: its profile takes into account the incidence of the loads of the beams of the mezzanine. On the other hand the high voussoirs of the keystone elevates the curve of pressures and allows its conduction within the safe zone at the point of rupture.

From the point of view of the functionality, the elevation from the ground plane of the starting points of the arches in 65 cm. allowed to include in the vain of the arc the space necessary for the passage of large carriages. In addition the layout of the arches with the widened upper part allowed the necessary space to lodge the initial sculptures of wrought iron of the owner in the tympans.

It was not possible to find a geometric or static reason that defined the profile of the arches. Only was tested as a conjecture the existence of some formal relations between a catenary and the upper section of the arc near the keystone, by interpreting the first one as the extrados of the arc curve.

Some important data arise from the study of the constructive configuration of Palau Güell. It is interesting to note that this seems to be the first occasion when Gaudí used metallic profiles for the construction of the slabs instead of the wooden beams he had used in all his previous works. It was then an extremely innovative material in residential architecture despite having already years of use in engineering and monumental works. Churches with exposed metal structures had already been built, such as Saint-Augustin in Paris, work of Víctor Baltard (1860-1871).

It should be noted that the first structural architectural use of steel dates from 1884-1885, in the office building of the Home Insurance Company of Chicago projected by William Le Baron Jenney, where the metallic structure was partially of this metal, only in the higher plants.[39] In 1889 when the Palau Güell was already built, the same Le Baron

Jenney will use for the first time a completely steel supporting structure in the second Leiter building, also in Chicago. The same year two great works destined to the propaganda of the possibilities of metallic structurae would be built at the Universal Exhibition of Paris: the Galerie des Machines and the Eiffel Tower.

These data only demonstrate the enormous vocation of modernity that Güell –and Gaudí- instilled in his palace, as a constructive manifesto, which explains his detachment for some traditional forms in favor of others that were capable of transmitting the values he wished to represent. In example the metallic structural elements that were visible in the great hall of the building, something completely unusual at the time and almost an anticipation of the constructive expressionism that would be proclaimed decades later. This could also explain the unusual shape of the arches of the main portals, although in this case we have seen that its shape does not arise from the dictation of mechanics.

Until it can be proved otherwise, it seems more the result of chance than of a project search the fact that its profile resembles a Rankine curve. In any case, as stated, a risky hypothesis can be proposed on the way in which this shape was drawn, but not responding to reasons of stability.

Gaudí was an architect who assembled as few all the components of architecture. It is attributed to him the expression "it is the aesthetic and not the static what determines the forms", apparently in discrepancy with an article by his collaborator Joan Rubió.[40] The study of his works shows that everything in them has a reason and everything responds to a general principle of unity that, to express it in a way already incorporated into gaudinian terminology, is the product of an organic concept of architecture.

Here seems to prevail as a general idea of the conception of this palace the vocation to pay homage to the modernity incarnated in its owner, Eusebio Güell. It is in this field that this hypothesis on the motivations of the shape of the arches of the facade of the Palau Güell is exposed.

NOTES

[1] Partially published in SCIORTINO, BIBIANA; ANDRUET, MARIO. Gaudí y la geometría. Gaudí Club. Barcelona. Nº 14, July 1995; Nº 15, Decembere 1995; Nº 16, April 1996; Nº 17, Special Edition, September 1996; and Nº 18, March 1997.

[2] Blas Herrera, Applied Geometry group, Departament d'Enginyeria Informàtica i Matemàtiques (DEIM); Albert Samper, Escola Tècnica Superior d'Arquitectura, and Genaro González, Departament d'Enginyeria Mecànica. Universitat Rovira i Virgili. Tarragona.

[3] SAMPER, ALBERT; GONZÁLEZ, GENARO; HERRERA, BLAS (2017).

[4] SAMPER, ALBERT; GONZÁLEZ, GENARO; HERRERA, BLAS (2016).

[5] RANKINE, WILLIAM JOHN MACQUORN. (1843). "Description of a method of laying down railway curves on the ground". Minutes of the Proceedings of the Institution of Civil Engineers. 2. Pp. 108-111.

[6] RANKINE, WILLIAM JOHN MACQUORN. A manual of applied mechanics. Griffin. London. 1858. French translation of the 7ª ed. by A. Vialay: Manuel de mécanique appliquée. Dunod. París. 1876.

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[7] BASSEGODA NONELL, JOAN. El Gran Gaudí. Ausa. Sabadell. 1989. P.514.

[8] Ibid. P.288.

[9] GONZÁLEZ MORENO-NAVARRO, JOSÉ LUIS; CASALS BALAGUÉ, ALBERT (2005).

[10] RÀFOLS FONTANALS, JOSEP (1929).

[11] VIOLLET-LE-DUC, EUGÈNE EMMANUEL. Dictionnaire raisonné de l'architecture française du XI au XVI siècle.

Libraires-Imprimeries Réunies. Paris 1854-68. Volume IV. P. 56.

[12] RANKINE, WILLIAM JOHN MACQUORN. (1876) P. 203.

[13] Ibid. Pp.219-221.

[14] RANKINE, WILLIAM JOHN MACQUORN. (1862) Pp. 202-203. (1876). Pp.190-191 y 200-201.

[15] RANKINE, WILLIAM JOHN MACQUORN. (1862) P.203.

[16] MOSELEY, HENRY. (1843). P. 380.

[17] Ibid. P. 432.

[18] RANKINE, WILLIAM JOHN MACQUORN. (1862) Pp. 202-203. (1876). Pp.190-191 y 200-201.

[19] RANKINE, WILLIAM JOHN MACQUORN. (1862). Pp.186-188.

[20] This issue was treated throughout the 19th century by various researches as Henry Moseley, Edouard Mery or Karl Culmann.

[21]RANKINE, WILLIAM JOHN MACQUORN. (1862). Pp. 416-417. (1876). Pp.256-257.

[22] Ibid. P. 437.

[23] RUBIÓ BELLVER, JOAN (1913).

[24] Exists a reconstruction in the Museu de Les Aigües in Cornellà de Llobregat.

[25] SCIORTINO, BIBIANA (2004). P. 113.

[26] GAUDÍ, ANTONIO. Manuscrito de Reus. 1876-1879.

[27] BERGÓS MASSÓ, JOAN. (1953) P.258.

[28] BASSEGODA NONELL, JOAN (2003).

[29] RONDELET, JEAN BAPTISTE. (1802). Pp. 70-74. Explains how he used a model to design the catenary arc that supports the colonnade of the dome and the lantern of the church of Sainte Génévieve in Paris. He refers to the catenary as the appropriate shape to form arches and vaults and exposes the graphic method that allows to obtain the points of the curve.

[30] RÀFOLS FONTANALS, JOSEP (1929).

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ANNEX I

ANALYSIS OF THE CHARGES THAT AFFECT THE ARCHES

The arches of the Palau Güell portals receive the load distributed from the façade and slabs to the lower level of the slab on the ground floor plus the load of the wall section between the floor on the ground floor and the intrados of the arc (the spandrels). In addition, in the corresponding section, they support the loading of the beams of the mezzanine.

DATA FOR THE CALCULATION
Extracted from the reports of the restorers of the building.
Slabs:
IPN 16: 17,90kg/ml
For a separation between axes 0,65: 1,52 ml = 27,1 kg/m².
Mortar filling: 1.600 kg/m³
Brick and lime mortar masonry: 1.700 kg/m³
Tile floor with mortar: 20 kg/cm. thickness
Wood ceiling included structure: 25 kg/m²
Overloads: An overload of 100 kg/m² is adopted as static charges (furniture and objects). Dynamic overloads are not considered (people).
Walls: limestone wall: 2450 kg/m³

1. LOADS OF THE SLABS
Roof
Iron beams.......................................27,10 kg/m²
Mortar filling: 1.600 kg/m³
x 0,20 x 0,30/2 x 3..........................144,00 “
Brick vaults: 20 x 2,5...................... 50,00 “
Board: 20 x 4 x 3............................240,00 “
Floor: 20 x 2,5...................................50,00 “
Wooden ceiling..................................30,00 “
Total...541,10 kg/m²
Surface of incidence for m. on facade: 2,60 m²
541,10 kg/m² x 2,60 m² ....1.406,86 kg

Third floor
Wooden structure......................245,00 kg/m²
Floor: 20 x 15.............................300,00 “
Overload......................................100,00 “
Total...645,00 kg/m²
Surface of incidence for m. on facade: 2,60 m²
645,00 kg/m² x 2,60 m² ....1.651,00 kg

Second floor
Wooden structure ....315,00 kg/m²
Overload:.....................................100,00 “
Total...415,00 kg/m²
Surface of incidence for m. on facade: 2,60 m²
415,00 kg/m² x 2,60 m² ....1.079,00 kg

First floor
Iron beams ....27,10 kg/m²
Mortar filling: 1.600 kg/m³
x 0,20 x 0,30/2 x 3........................144,00 “
Brick vaults: 20 x 2,5.......................50,00 “
Deck: 20 x 4 x 3..............................240,00 “
Floor: 20 x 2,5..................................50,00 “
Wooden ceiling ....100,00 “
Overload:.......................................100,00 “

Total...711,10 kg/m²
Surface of incidence for m. on facade: 2,60 m²
711,10 kg/m² x 2,60 m² ....1.848,60 kg

Total load slabs and roof:
1.406,86 + 1.651,00 + 1.079,00
+ 1.848,6.......................................5.985,46 kg/ml.

2. FACADE WALL
Limestone wall 0,70 m. thickness
Weight: 2.450 kg/m³
Height from lower level of slab Ground floor: 14.00 m.
Due to the many windows a reduction coefficient will be applied:
Wall surface: 23,50 x 14,00= 329,00 m2
Windows surface: 3,60 x 14 + 3,25 x 6 + 5,15 + 2,75 x 14 + 3,70 + 7,85 + 2,40 x 5 + 12,80 + 4,70 x 2 + 0,70 x 2 + 4,40 x 2 + 7,10 x 3= 196,80 m2
Effective Surface of the wall: 329,00 – 196,80 = 132,20 m2
Stone cantilevers surface: (2,10 + 3,20 + 19,50) x 1,00= 24,80 m2
Cantilevers weight: 24,80 x 0,25 x 2.450 kg/m³= 15.376 kg
Weight of the Wall over arches keystones:
132,20 m2 x 2.450 kg/m³= 323.890 kg + 15.376 kg (cantilevers)= 339.266 kg
Incidence for m. on facade: 339.266 / 23,50 m= 14.436,85 kg
Incidence top:
(2.450 kg/m³ x 0,50 x 1,50 m.*) + 200 kg. (chimneys)=........2.037,50 kg/ml.
*Average of the height of the parapet
Total facade Wall load
14.436,85 + 2.037,50= 16.474,35 kg/ml.

TOTAL DISTRIBUTED LOAD AT LOWER LEVEL OF THE SLAB ON THE GROUND FLOOR:
5.985,46 + 16.474,35........22.459,81 kg/ml.

3. LOADS OF THE MEZZANINE
(apply at 1,65 m. from the axis)
Mezzanine floor
Iron beams ....27,10 kg/m²
Mortar filling: 1.600 kg/m³
x 0,20 x 0,30/2 x 3............................144,00 “
Brick vaults: 20 x 2,5..........................50,00 “
Deck: 20 x 4 x 3.................................240,00 “
Floor: 20 x 2,5.....................................50,00 “
Overload:...100,00 “
Total..611,10 kg/m²

Lateral beam
Load of the slab:1,30 x 0,70
x 611,10 kg/m²………………………….556,10 kg.
Partition load: 0,15 x 3,00
x 1.800 kg/m³…………………………….810,00 kg.
Own load: 0,30 x 0,60 x 0,.70
x 2.450 kg/m³…………………………….308,70 kg.
Total load of exterior beam= 1.674,80 kg.

Central beam
Load of the slab:1,20 x 0,90
x 611,10 kg/m²…………………………...659,88 kg.
Partition load: 0,15 x 3,00
x 1.800 kg/m³…………………………….810,00 kg.
Own load: 0,30 x 0,60 x 0,90
x 2.450 kg/m³…………………………….396,09 kg.
Total load of interior beam = 1.866,78 kg.

4. SPANDRELS AND OWN LOAD OF THE ARCHES

This section is measured from the lower floor plan of the Ground Floor to the intrados of the arc. As a method of calculation to discount the recess of the intrados of the arc, an average between the outer face and the inner face in each section will be considered.

It will be analyzed the load on a half-arc between the key and the point of rupture. This is located on the joint at an angle of 60° from the axis of the arc. The half-arc will be divided into 12 sections of 0.20 m. each.

Material: limestone wall (2450 kg/m³). Thickness: 0,70 m.

Section 1
Exterior face: 0,20 x (1,05 + 1,07) / 2= 0,212 m²
Interior face: 0,20 x (1,55 + 1,57) / 2= 0,312 m²
Volume: (0,212 + 0,312) / 2 x 0,70= 0,183 m³
Weight: 2450 kg/m³ x 0,183 m³= 449 kg
Section 2
Exterior face: 0,20 x (1,07 + 1,10) / 2= 0,217 m²
Interior face: 0,20 x (1,57 + 1,62) / 2= 0,319 m²
Volume: (0,217 + 0,319) / 2 x 0,70= 0,188 m³
Weight: 2450 kg/m³ x 0,188 m³= 460 kg
Section 3
Exterior face: 0,20 x (1,10 + 1,15) / 2= 0,225 m²
Interior face: 0,20 x (1,62 + 1,70) / 2= 0,332 m²
Volume: (0,225 + 0,332) / 2 x 0,70= 0,195 m³
Weight: 2450 kg/m³ x 0,195 m³= 478 kg
Section 4
Exterior face: 0,20 x (1,15 + 1,23) / 2= 0,238 m²
Interior face: 0,20 x (1,70 +1,80) / 2= 0,350 m²
Volume: (0,238 + 0,350) / 2 x 0,70= 0,206 m³
Weight: 2450 kg/m³ x 0,206 m³= 504 kg
Section 5
Exterior face: 0,20 x (1,23 + 1,33) / 2= 0,256 m²
Interior face: 0,20 x (1,80 +2,01) / 2= 0,381 m²
Volume: (0,256 + 0,381) / 2 x 0,70= 0,223 m³
Weight: 2450 kg/m³ x 0,223 m³= 546 kg
Section 6
Exterior face: 0,20 x (1,33 + 1,60) / 2= 0,293 m²
Interior face: 0,20 x (2,01 +2,41) / 2= 0,442 m²
Volume: (0,293 + 0,442) / 2 x 0,70= 0,257 m³
Weight: 2450 kg/m³ x 0,257 m³= 630 kg
Section 7
Exterior face: 0,20 x (1,60 + 1,75) / 2= 0,335 m²
Interior face: 0,20 x (2,41 +2,99) / 2= 0,540 m²
Volume: (0,335 + 0,540) / 2 x 0,70= 0,306 m³
Weight: 2450 kg/m³ x 0,306 m³= 750 kg
Section 8
Cara exterior: 0,20 x (1,75 + 2,08) / 2= 0,383 m²
Interior face: 0,20 x (2,99 +3,98) / 2= 0,697 m²
Volume: (0,383 + 0,697) / 2 x 0,70= 0,378 m³
Weight: 2450 kg/m³ x 0,378 m³= 926 kg
Section 9
Exterior face: 0,20 x (2,08 + 2,55) / 2= 0,463 m²
Interior face: 0,20 x (3,98 + 5,69) / 2= 0,967 m²
Volume: (0,463 + 0,967) / 2 x 0,70= 0,501 m³
Weight: 2450 kg/m³ x 0,501 m³= 1.226 kg
Section 10
Exterior face: 0,20 x (2,55 + 3,35) / 2= 0,590 m²
Interior face: 0,20 x (5,69 + 6,35) / 2= 1,204 m²
Volumen: (0,590 + 1,204) / 2 x 0,70= 0,628 m³
Weight: 2450 kg/m³ x 0,628 m³= 1.538 kg
Section 11
Exterior face: 0,20 x (3,35 + 4,87) / 2= 0,822 m²
Interior face: 0,20 x 6,35 = 1,270 m²
Volume: (0,822 + 1,270) / 2 x 0,70= 0,732 m³
Weight: 2450 kg/m³ x 0,732 m³= 1.794 kg
Section 12
Exterior face: 0,20 x (4,87 + 6,35) / 2= 1,122 m²
Interior face: 0,20 x 6,35 = 1,270 m²
Volume: (1,122 + 1,270) / 2 x 0,70= 0,837 m³
Weight: 2450 kg/m³ x 0,837 m³= 2.051 kg

5. LOADS ON THE ARCHES

Section 1
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ……………………………….449 kg
Total: ……………………………………………4.941 kg
Section 2
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ……………………………….460 kg

Total: ……………………………………………4.952 kg

Section 3
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ………………………………478 kg
Total: ……………………………………………4.970 kg

Section 4
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ………………………………504 kg
Total: ……………………………………………4.996 kg

Section 5
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ………………………………546 kg
Total: ……………………………………………5.038 kg

Section 6
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ………………………………630 kg
Total: ……………………………………………5.122 kg

Section 7
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ………………………………750 kg
Total: ……………………………………………5.242 kg

Section 8
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ………………………………926 kg
Load beams mezzanine: 1.675 (lateral)/ 1.867 kg. (central)
Total: 7.093 (ext.) / 7.285 kg

Section 9
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: …………………………………1.226 kg
Total: ………………………………………………5.718 kg

Section 10
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: …………………………………1.538 kg
Total: ………………………………………………6.030 kg

Section 11
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: …………………………………1.794 kg
Total: ………………………………………………6.286 kg

Section 12
Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: …………………………………2.051 kg
Total: ………………………………………………6.543 kg

ANNEX II

ANALYSIS OF THE CHARGES THAT AFFECT THE ARCHES ACCORDING TO THE REJECTED PROJECT WITH REDUCED ARCHES

As in the case of the arches that were finally built, they receive the load distributed from the façade and slabs to the lower level of the slab over the Ground Floor plus the load of the wall section between the slab over the Ground Floor and the intrados of the arch. Also in the corresponding section they support the loading of the beams of the mezzanine.

The load distributed to the lower level of the slab on the ground floor and the loads of the mezzanine are the same as in the previous case. We'll calculate the loads corresponding to the section between the slab of the Ground Floor and the intrados of the arc. The drawing shows more regular voussoirsthan in the constructed arches, with intrados with no recess.

The half-arc was divided into 11 sections of 0.20 m. each.

Section 1
0,20 x (2,63 + 2,64) /2 x 0,70= 0,369 m³
Weight: 2450 kg/m³ x 0,369 m³= 904 kg

Section 2
0,20 x (2,64 + 2,66) /2 x 0,70= 0,371 m³
Weight: 2450 kg/m³ x 0,371 m³= 909 kg

Section 3
0,20 x (2,66 + 2,68) /2 x 0,70= 0,378 m³
Weight: 2450 kg/m³ x 0,378 m³= 915 kg

Section 4
0,20 x (2,68 + 2,71) /2 x 0,70= 0,377 m³
Weight: 2450 kg/m³ x 0,377 m³= 924 kg

Section 5
0,20 x (2,71 + 2,76) /2 x 0,70= 0,383 m³
Weight: 2450 kg/m³ x 0,383 m³= 938 kg

Section 6
0,20 x (2,76 + 2,81) /2 x 0,70= 0,390 m³
Weight: 2450 kg/m³ x 0,390 m³= 956 kg

Section 7
0,20 x (2,81 + 2,87) /2 x 0,70= 0,398 m³
Weight: 2450 kg/m³ x 0,398 m³= 974 kg

Section 8
0,20 x (2,87 + 2,95) /2 x 0,70= 0,407 m³
Weight: 2450 kg/m³ x 0,407 m³= 998 kg

Section 9
0,20 x (2,95 + 3,04) /2 x 0,70= 0,419 m³
Weight: 2450 kg/m³ x 0,419 m³= 1.027 kg

Section 10
0,20 x (3,04 + 3,11) /2 x 0,70= 0,431 m³
Weight: 2450 kg/m³ x 0,431 m³= 1.054 kg

Section 11
0,20 x 3,11 x 0,70= 0,435 m³

Weight: 2450 kg/m³ x 0,435 m³= 1.067 kg

LOADS ON THE ARCHES

Section 1

Wall load: 22.460 x 0,20 = 4.492 kg.
Spandrel load: ……………………………..904 kg
Total: ……………………………………….5.396 kg

Section 2

Wall load: 22.460 x 0,20 = . 4.492 kg.
Spandrel load: ……………………………..909 kg
Total: ……………………………………….5.401 kg

Section 3

Wall load: 22.460 x 0,20 = 4.492 kg.
Spandrel load: ……………………………..915 kg
Total: ……………………………………….5.407 kg

Section 4

Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ……………………………..924 kg
Total: ……………………………………….5.416 kg

Section 5

Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ……………………………..938 kg
Total: ……………………………………….5.430 kg

Section 6

Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ……………………………..956 kg
Total: ……………………………………….5.448 kg

Section 7

Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ……………………………..974 kg
Total: ……………………………………….5.466 kg

Section 8

Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: ……………………………..998 kg
Total: ……………………………………….5.490 kg

Section 9

Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: …………………………..1.027 kg
Load beams mezzanine: 1.675 (lateral)/
1.867 kg. (central)
Total: 7.194 (ext.) / 7.386 kg

Section 10

Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: …………………………..1.054 kg
Total: ……………………………………….5.546 kg

Section 11

Wall load: 22.460 x 0,20 = 4.492 kg
Spandrel load: …………………………..1.067 kg
Total: ……………………………………….5.559 kg